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SCHWARZSCHILD PROBLEM FOR A METRIC WHOSE SPATIAL PART IS PURELY EUCLIDEAN

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In the course of investigation of motion of a material medium, use is often made of curvilinear coordinates. In such a case, the metric of curved space is given by the interval

$$-ds^2 = g_{ik}\,dx^i\,dx^k$$

where g_{ik} are the components of the metric tensor.

In the case of motions possessing central symmetry (after some coordinate transformation, should it be necessary), the usual interval is given in the form [1] of a Schwarzschild metric

$$-ds^2 = -e^{\nu}c^2dt^2 + e^{\lambda}\,dr^2 + r^2(d\theta^2 + \sin^2\theta\,d\varphi^2 \qquad (\nu = \nu(t, r),\ \lambda = \lambda(t, r))$$

However, in a number of gas-dynamics problems, a different metric is found to be of use, in which the spatial part of the interval is Euclidean

$$-ds^2 = -e^{\nu}c^2\,dt^2 + 2e^{1/2\mu}\,cdt\,dr + dr^2 + r^2(d\theta^2 + \sin^2\theta\,d\varphi^2) \qquad (\mu = \mu(t, r))$$

With this condition adopted, the investigation of the motion of gas in the gravity field becomes considerably simplified. Purely spatial metric will still be a curve

$$dl^2 = (1 + e^{\mu-\nu})\,dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)$$

and so will geodesic trajectories.

The above metric can be obtained directly from the Schwarzschild's metric by putting $e^{\nu} = 1 - r_0/r$, $\lambda = -\nu$, and using the transformation

$cdt = cdt_1 + D(r)\,dr$. This gives

$$-ds^2 = -(1 - r_0/r)\,c^2\,dt^2 \pm 2\,(r_0/r)^{1/2}\,cdtdr + dr^2 + r^2\,(d\theta^2 + \sin^2\theta\,d\varphi^2) \qquad (1)$$

where $r_0 = 2GM/c^2$ is the gravitational radius.

This metric is found convenient when quantization is to be performed in a strong Schwarzschild field. We should note that neither the above metric nor the Finkelstein metric possess the singularities at $r = r_0$. When $r_0 > r$, the metric (1) becomes space-like.

Let us consider the motion of a representative point in the equatorial plane ($\theta = \frac{1}{2}\pi$) of a gravitating body with the metric (1). Using the equation of a geodesic, we have

$$\frac{d^2\varphi}{ds^2} + \frac{2}{r}\frac{dr}{ds}\frac{d\varphi}{ds} = 0$$

$$\frac{d^2r}{ds^2} + \frac{r_0}{2r^2}\left(1 - \frac{r_0}{r}\right)\left(\frac{cdt}{ds}\right)^2 + \frac{r_0}{r^2}\left(\frac{r_0}{r}\right)^{1/2}\frac{cdt}{ds}\frac{dr}{ds} - \frac{r_0}{2r^2}\left(\frac{dr}{ds}\right)^2 - r\left(1 - \frac{r_0}{r}\right)\left(\frac{d\varphi}{ds}\right)^2 = 0 \qquad (2)$$

$$\frac{d^2(ct)}{ds^2} + \frac{r_0}{2r^2}\left(\frac{r_0}{r}\right)^{1/2}\left(\frac{cdt}{ds}\right)^2 + \frac{r_0}{r^2}\frac{cdt}{ds}\frac{dr}{ds} + \frac{r_0}{2r^2}\left(\frac{r_0}{r}\right)^{1/2}\left(\frac{dr}{ds}\right)^2 - r\left(\frac{r_0}{r}\right)^{1/2}\left(\frac{d\varphi}{ds}\right)^2 = 0$$

(1) and (2) yield a well-known equation of the orbit of a particle

$$\frac{d^2\xi}{d\varphi^2} + \xi = \frac{1}{p}\left(1 + \frac{3GMp}{c^2}\,\xi^2\right) \qquad (3)$$

which has a solution [2]

$$\xi = \frac{1}{p}\,\{1 + (3 + e^2)\,A + e^2A\sin^2\varphi + e\cos[\varphi\,(1 - 3A)]\} \qquad \left(\xi = \frac{1}{r},\ A = \frac{GM}{pc^2}\right)$$

where p is the parameter and e is the eccentricity of the orbit. We should note that Expressions

$$\frac{d\varphi}{ds} = (A)^{1/2}\frac{p}{r^2}, \qquad \frac{dr}{ds} = -(A)^{1/2}\,\{Ae^2\sin 2\varphi - e\,(1 - 3A)\sin[\varphi\,(1 - 3A)]\}$$

$$\frac{cdt}{ds} = \frac{eA\sqrt{2\,(1 + e\cos[\varphi\,(1 - 3A)]\}}\,\sin[\varphi\,(1 - 3A)] \pm \sqrt{A\,(e^2 - 1) + 1}}{1 - 2A\,\{1 + e\cos[\varphi\,(1 - 3A)]\}}$$

yield the values of the 4-velocity components of the particle, which are different from zero.

Also, we can easily see that time has no singularities. The latter is important in analysis of motion of a particle in the Schwarzschild field.

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